Past performance is not a reliable indicator of future results or a guarantee of future returns.

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I. What is Volatility? (Section 1/2)

In Finance, volatility is a measure of the markets variability. The higher the volatility, the stronger the price of an asset changes in a short period of time. In other words, it describes how (un)stable a market is, the “speed of the market”, but not its direction, neither its performance.

- Realised versus implied volatility

When talking about volatility, it is essential to differentiate between “realised volatility” and “implied volatility”.

- Realised Volatility is an ex-post indicator of the markets variability. It is the volatility observed in the market. Realised volatility can be defined as “A statistical measure of the dispersion of returns for a given security or market index”.

- Implied Volatility is an ex-ante risk indicator. It reflects the market expectations of an asset’s future fluctuations. It is inferred from the price of options traded in the market.

In this first chapter we will focus on realised volatility. Implied volatility will be discussed in the next part.

- Realised volatility: a measure of the markets past variability

The realised volatility of a historical data series can be measured by using the standard deviation of returns (measure of the dispersion of returns versus the mean return) of an asset for a specific time interval (e.g. daily) over a given period (e.g. monthly or annually):

$$\sigma(x) = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}$$

With:

- $\sigma(x)$: standard deviation of returns
- $x_i$: daily return at time $i$
- $\bar{x}$: mean return
- $n$: number of days

The following graph shows a share price, which rises 5% in 14 days. The mean return is 0.35% per day. The daily returns diverge from the mean return by +/- 1%. The standard deviation, i.e. the daily realised volatility, is 1%.
Volatilities are annualised for comparison purposes. Annualising volatility is fairly simple as it is proportional to the square root of time, e.g.: A daily volatility of 1% equals an annual volatility of 15.9%, as 1% x \(\sqrt{252}\) = 1% x 15.9 = 15.9%, assuming there are 252 trading days in one year.

A one year volatility of 15.9% means that the return of an asset diverges by 1% up or down from its mean return every day over one year.

A one month annualised volatility of 40% – e.g. level reached by the 20-day realised volatility on the Euro Stoxx 50 in Summer 2015 – means that the return of an asset diverges by approx. 2.5% (= 40% / 15.9) up or down from its mean return every day over one month. This is illustrated by the graphs below.

- **Volatility does not reflect a direction, nor a performance**

Volatility does not indicate whether the price of an asset moves up or down. It just reflects the dispersion. Mathematically this can be explained by the fact that when calculating the standard deviation all differences versus the mean are squared. Hence the impact will be the same whether the difference versus the mean is positive or negative.

The graphs below show three return patterns over a 22-day period (i.e. one month). In the first graph the return over the entire period of all curves is 0%. It is +1.5% in the second and -1.5% in the third.

When the daily return is constant (dotted black line), the realised volatility is 0, irrespective of the direction: flat, up or down. When the daily return diverges on average 1% (dark blue line) from its mean, volatility is at 16%. It is at 40% for a daily divergence of 2.5% (light blue line).

**Correlation between volatility and its underlying market**

Although the volatility itself does not reflect the up or down trend of the underlying asset price, the graph below suggests that in practice (as opposed to the theory) there is a negative correlation between the equity market trend and the evolution of equity market volatility.
Indeed the graph reflects that often volatility rises when the Euro Stoxx 50 drops, and falls when the equity index rises.

Realised equity volatility is negatively correlated to the underlying equity market. This negative correlation comes from the fact that equity market drawdowns driven by fear and risk aversion create on average larger and more abrupt swings than upward moves driven by investor confidence. Therefore extreme negative equity returns lead to more volatility than positive ones.

This negative correlation does not apply 100% of the time. The Nikkei index for example has seen periods of rising volatility during bull markets, as was the case in the 1st quarter of 2013.

**Quiz**

What is the origin of the word « volatility »?

Volatility comes from „volatilitas“ in Latin, which means: flying, equipped with wings, passing through the air.

**Sources**
2. Investopedia.com
II. What is Volatility? (Section 2/2)

The first chapter of our Volatility academy was dedicated to realised (=historical) volatility, which measures the dispersion of an asset's past returns around its average. In this chapter we will discuss the basics of implied volatility ("IV").

Implied volatility reflects the market's expectations of future price fluctuations of an asset. Alike historical volatility, IV does not indicate a direction neither a performance. It is an estimate of the distribution of future returns. Both realised and implied volatility are strongly correlated.

- Volatility through pinball

Implied volatility reflects expectations of future return distribution. As there is no guarantee that the prediction will come true, implied volatility is probability.

. To better understand why implied volatility is probability and used to predict future return distribution but not the direction, we will take the example of a pinball maze. Consider the share price as a ball that encounters at each nail a 50% chance to move to the right, and a 50% chance to move to the left. The ball adopts a random walk (figure 1). A random walk consists of a succession of random steps according to a set of probabilities. E.g. the path taken by a ball dropped on a pin maze, of a molecule traveling through the air or possibly the price evolution of a fluctuating share.

. Balls tend to fall near the middle, with a decreasing number of balls dropping away from the centre. We obtain a normal distribution (figure 2). In a nutshell, a random variable has a normal distribution if “its possible values fall into a smooth curve with a bell-shaped, symmetric pattern, meaning it looks the same on each side when cut down the middle”.

The following illustration aims to bridge the gap between the bell-shaped distribution of the pinball and the expected returns of an asset. The horizontal axis represents the daily returns of the asset. The further the balls move away from the mean, the more extreme the value is i.e. the daily return diverges from the mean return. The red balls in the graph below represent the extremes, which are referred to as distribution tails.
What drives Implied Volatility?

Investor sentiment, option prices and thus implied volatility are all driven by the same factors, which impact financial markets in general: news flow regarding the underlying asset as well as supply and demand. In the case of options on equity indices major positive and negative impacts will be driven by:

- macro-economic news and monetary policy;
- demand for protection and/or for upside exposure via put/call options, flows from structured products.

While markets are aware of some of these factors and expecting them (“known unknowns”), others can hit markets by surprise (“unknown unknowns”).

Implied Volatility: a key variable in option pricing

The price of an option (or option premium) has two components:

- **Intrinsic value** results from the current price of an asset and the strike price of an option. Let’s consider a call option which allows to buy an asset at a price of 500 (= strike price), while the underlying asset is worth 600. The intrinsic price of the call option will be 100.
- **Time value** represents the value of the time left until the option expires. This is measured with option models.

The Black & Scholes model is the world’s best-known model to price options. It allows to calculate the theoretical price of a call or put option. The model takes into account the current asset price, the option strike price, the time until the option expiry, the interest rate, and the implied volatility. Note: for simplicity we ignore any potentially expected dividend payments.

The graph below illustrates how the price of an option varies according to the volatility assumption. We have used a 1-year at-the-money put option on the S&P 500.

The option price (=premium) is expressed in percentage of the spot of the underlying asset. When the option premium is at 7.1%, the IV is at 16.4%. In November 2008, IV increased to 50%.

Source: Amundi, Bloomberg as of 30th May 2016, Bloomberg function “OV”
Implied volatility is the only **unknown or potentially uncertain variable in the pricing model.** Therefore, when the price of the option is known, then the implied volatility can be derived from the equation.

- **Implied Volatility indices used as risk indicator**

In the same way as realised equity volatility is normally negatively correlated to the underlying equity market, IV tends to rise when investor fear increases as a consequence of greater market uncertainty. As a consequence implied volatility is commonly used as a risk indicator, though it is not the only one.

Implied volatility can easily be monitored via short term indices. The most common index is the VIX, which reflects the 1-month implied volatility for options on the S&P 500 index. It is not a directly investible index, though long/short positions can be taken via futures. Equivalent indices exist for other major indices: VSTOXX for the Euro Stoxx 50 or VNKY for the Nikkei 225.

The chart shows the evolution of the VIX and of the 20 day realised volatility of the S&P 500. Not surprisingly both curves are very similar. On average implied volatility is above realised volatility, reflecting the risk premium of the asset class, in other words the significantly greater risk taken by option sellers (i.e. that volatility surges) than by option buyers, whose losses are limited to the option premium paid.

Source: Bloomberg as of 17 March 2016

- **Quiz**

What is the highest level ever reached by the VIX index, and what does that mean in terms of expected dispersion around the mean return?

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**Sources**

3. “There are known knowns; there are things we know that we know. There are known unknowns; that is to say there are things that, we now know we don't know. But there are also unknown unknowns – there are things we do not know, we don't know”. Donald Rumsfeld, United States Secretary of Defense, news briefing on 12 Feb. 2002
III. Volatility Term structure, Smile/skew and Surface

In the previous chapters, we discussed realised and implied volatility (IV). In this part we will explore how implied volatility can change according to an option’s maturity and strike level and what the spreads between different levels can reveal.

■ Volatility term structure

Implied volatility reflects expectation of future volatility. As market participants’ perception of future uncertainty can change according to the time horizon, IV can vary according to the option maturity. For example, if the market faces short-term risks, implied volatility of short-term options will be higher than the IV of longer-term options. Thus, the term structure of volatility “looks at the implied volatilities for a particular underlying and strike across different maturities”1.

The term structure curve can be described as either steep, flat or inverted. “Normal markets” are reflected by a steep upward sloping term structure, as uncertainty increases with time.

To better understand, let’s look at the S&P 500 volatility term structure of at-the-money (ATM) options during the summer 2015 and February 2016:

. On 16th July 2015: the term structure was steep, reflecting a “normal environment”, with no major short-term threats. Mid-term volatility was higher as markets expected inter alia a rate hike by the FED prior to year end.
. On 24th August 2015 “Black Monday”: the term structure was inverted following the market correction triggered by the surprise yuan devaluation. The market perceived significant short-term uncertainty, such that short-term IV was higher than for longer-term options, which had also risen but to a lesser extent.
. On 15th September 2015: as the crisis had calmed down, short-term IV retraced whereas long-term IV was almost unchanged. This led to a flat term structure. Uncertainty remained, though the imminent risks had receded so that there was no volatility premium associated with time.
. On 11th February 2016: renewed concerns over Chinese growth, coupled with an oil price drop triggered a new equity market drawdown and volatility spikes. Interestingly, on the short end, IV remained below the levels reached the previous summer. However, an upward shift of mid-term IV reflected deeper concerns over long-term global growth prospects.

■ Volatility smile

The volatility smile looks at implied volatilities across different strikes for a same expiry date. It reflects the expected variations in implied volatility according to the market moves. The volatility smile reflects the supply and demand for options for hedging purposes and/or upside exposure. For example, when looking at the 1yr expiry, if the market drops rapidly by 10%, the 90% option will give the expected level of 1yr ATM IV.

For the sake of clarity, we want to highlight that IV is the same for call and put options with the same strike and expiry date. This is underpinned by the put/call parity (assumption that the sum of a long call and short put with the same strike and maturity is equivalent to a forward contract). Indeed, IV reflects the anticipation of future market variability and risk expectations, which do not depend whether one prices a call or a put option.
When plotting implied volatilities against strike prices, one should obtain a U-shaped curve, reason why we call it the volatility smile. **Implied volatility increases as the option is increasingly in-the-money (ITM) or out-of-the-money (OTM).**

Volatility smiles are common for foreign exchange options when demand for two currencies is balanced. However, the “smile” is not the only possible pattern for a volatility curve. Indeed it can differ according to asset classes, market environment and most importantly supply and demand for options with different strike levels.

### Volatility skew

For equities, we talk about a **volatility skew** as the curve is normally downward sloping. This shape is the consequence of the negative correlation between implied volatility and the underlying equity market. Equity market drawdowns create on average larger and more abrupt swings than upward moves.

Investors are willing to pay more for protection against market downturns than for upside participation. As sellers of downside protection can face significant losses, they ask for a **volatility risk premium**. When investors believe that the market has little upside potential, they tend to sell OTM call options to afford this protection, accentuating the level of the skew. Generally the volatility skew is monitored by looking at the spread between the 90% put versus 110% call.

It can happen that the skew turns negative. In this case, the curve is upward sloping. Such negative skew was observed on the HSI equity index in Q2 2015, after the index rallied by 20% in 6 weeks and surged to a 7-yr high. As investors were expecting the index to continue to increase with a higher probability, there was a greater demand for OTM calls than for OTM puts.

While an upward sloping skew is rare for equities, it can be observed for options in the commodities market, when a shortage of a specific commodity is anticipated.
Skew for different expiry dates

Differences in skew levels for different maturities reflect different degrees of risk appreciation and thus for protection according to the time horizon. When comparing 1 month and 6 months skew on the S&P 500 since the beginning of 2016, it appears that both are following opposite trends. 1M skew is reducing as investors do not anticipate short-term risks and uncertainty to increase. The 6M skew in turn widened, as over the mid-term, several risk factors could impact financial markets: next FED hike, US elections, global growth, geopolitical risks around the World.

Volatility surface

When combining term structure and volatility skew into one three dimensional graph one obtains a volatility surface showing implied volatility as a function of strike and expiry date. This snapshot view is often used by traders to appreciate the cohesion between the various levels and to assess their risks.

Prior to the 1987 crisis (Dow Jones index down 22% in one day), equity traders used a perfectly flat volatility surface, in line with the constant volatility assumption of the Black & Scholes option pricing model. The crisis revealed that risks were not adequately remunerated. Since then traders demand higher premiums, reflected by higher IV, as risk increases.

Quiz

Over the last 3 years, which of the following equity indices features the highest 90%-110% skew for 6 months options and why: the S&P 500, the Euro Stoxx 50 or the Nikkei 225?

Sources

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IV. From risk indicator to investable asset class: how to get exposure to Volatility

In this chapter we will explain the most common instruments investors use to expose themselves to equity volatility.

#### Brief history of Volatility products

<table>
<thead>
<tr>
<th>Listed</th>
<th>Realised variance futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vanilla options</td>
<td>VIX futures</td>
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<td>VIX options</td>
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<td></td>
<td>VIX ETR</td>
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<td></td>
<td>VSTOXX futures</td>
</tr>
<tr>
<td></td>
<td>VNKY VIXB futures</td>
</tr>
<tr>
<td>OTC</td>
<td>Variance swaps</td>
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<tr>
<td></td>
<td>Forward variance</td>
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<td></td>
<td>Conditional variance</td>
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<td></td>
<td>Options on variance</td>
</tr>
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<td></td>
<td>Gamma swaps</td>
</tr>
</tbody>
</table>

Source: Barclays Equity Linked Strategies, February 2012

In 1973, standardized exchange traded options started to trade. The pricing of these options was made easier by the Black & Scholes formula (1970). The 1990s saw the explosion of OTC derivatives as hedge funds became increasingly popular. This led volatility to become an investable asset class. New volatility products such as variance & volatility swaps were created, and a decade later futures on volatility indices such as the VIX were launched. The financial crisis of 2008 significantly reduced the use of OTC products.

In this chapter, we will focus on listed vanilla options, variance swaps and VIX futures & ETPs. Each of these instruments can be used to gain either long or short exposure to volatility.

#### Listed vanilla options

One way to get exposure to volatility is to trade vanilla options. To be long (short) volatility, an investor needs to buy (sell) call or put options on the desired underlying (e.g. an equity index).

As explained in our 2nd chapter, the price of an option depends, among other factors, on the level of implied volatility. Options are thus primarily exposed to the underlying, volatility and interest rate moves. Looking at volatility on equity indices, in order to extract the volatility component, investors will hedge the equity risk and interest rate risk by trading futures on equity indices and interest rates.

**How to extract volatility from options?**

Source: Amundi Volatility Management team
Using options presents some benefits. The liquidity on index option markets (EUREX for the Euro Stoxx 50, CBOE for the S&P 500 and SGX & OSE for the Nikkei) is high and as opposed to other markets, it tends to increase in periods of market stress as investors use options to hedge their positions. In comparison to OTC instruments, liquidity is enhanced by the variety of investors (banks, asset managers, hedge funds, market makers, insurers…). Transaction costs and bid ask spreads are thus reduced.

However, investing in volatility via options requires some expertise. As mentioned above, options are exposed to market moves, i.e. the equity risk. To negate this effect, investors need to dynamically adjust their positions in order to be fully hedged against the equity risk. The volatility sensitivity is also moving with time and the underlying market level. Therefore, to keep a constant exposure, investors need to actively manage a portfolio of options and futures. Furthermore, the option price is affected by time decay and other parameters, which we will explore in the coming chapters.

### Variance Swaps

Variance swaps are OTC instruments which allow investors to exchange a realised variance over a specified period against a pre-agreed level of variance. Variance is the square of volatility.

The profit and loss depends directly on the difference between realised and implied variance. At maturity, the payoff of the variance swap is:

\[
Payoff = N \times (\sigma^2_{\text{realised}} - \sigma^2_{\text{strike}})
\]

Where \( N \) is the notional amount and \( \sigma^2 \) the variance of the underlying.

If the realised variance is superior to the strike level, the buyer of a variance swap will be in profit; and if the realised variance is below, the buyer will be in loss. **A buyer of a variance swap is therefore long volatility.** Variance swaps offer pure and direct exposure to volatility of an underlying asset without having to hedge the exposure to the underlying and to rates.

However, in order to hedge themselves, banks may replicate the variance swap by investing in options with different strikes. As replication is complex and not perfect, it leaves banks with some risks, which are reflected in wider bid/ask spreads and a medium liquidity. Moreover, as it is an OTC instrument, investors are exposed to counterparty risk and lower liquidity in periods of stress.

Please note, volatility swaps exist as well. However, it is easier for banks to replicate variance swaps through options than volatility swaps, if they need to hedge themselves. Variance swaps are thus more popular and more liquid than volatility swaps.

### VIX Futures

The VIX spot index, also known as the “fear gauge”, is based on the S&P 500. It reflects the market’s expectations for volatility over the next 30 days. It is calculated by looking at the IV of S&P 500 put and call options over a wide range of strike prices and short dated maturities. The VIX spot is a theoretical calculation and is not a tradable index. A number of instruments were developed to offer exposure: futures, options, ETPs (exchange traded products).
VIX futures allow to take views on the future level of the VIX spot index. These instruments do not need daily readjustments.

VIX futures are available for several maturities, as shown in the table. For example, the price of the VIX futures Dec’16 reflects the expected level of the VIX spot at the expiration date.

As futures have a defined maturity, investors will have to roll them before expiry in order to maintain a constant exposure. VIX futures thus require active management.

Prices of futures depend on the visibility of the market and the supply and demand for these contracts.

<table>
<thead>
<tr>
<th>Expiry</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIX spot</td>
<td>16.22</td>
</tr>
<tr>
<td>Jun ‘16</td>
<td>17.50</td>
</tr>
<tr>
<td>Jul ‘16</td>
<td>18.94</td>
</tr>
<tr>
<td>Aug ‘16</td>
<td>19.55</td>
</tr>
<tr>
<td>Sept ‘16</td>
<td>20.30</td>
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<tr>
<td>Oct ‘16</td>
<td>20.84</td>
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<td>Nov ‘16</td>
<td>21.00</td>
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<tr>
<td>Dec ‘16</td>
<td>21.05</td>
</tr>
<tr>
<td>Jan ‘17</td>
<td>21.75</td>
</tr>
</tbody>
</table>

Following the success of these instruments, futures on VSTOXX (equivalent of the VIX for the Euro Stoxx 50), VNKY (for the Nikkei 225) and VHSI were progressively launched. However the liquidity for the VSTOXX represents only 5% of the VIX market, and is almost inexistent on the VNKY and VHSI.

### Exchange Traded Products on VIX

ETPs on the VIX were launched to provide investors not allowed to trade futures (and derivatives) access to volatility. Furthermore they allow to avoid managing the roll, while still offering pure exposure to VIX futures. ETPs include ETNs (exchange traded notes) and ETFs (exchange traded funds). Each day, ETPs roll a part of their positions in order to maintain a 1-month exposure.

#### ETNs offering long volatility exposure

For example the VXX aims to provide the performance of the VIX short-term futures index.

However, depending on the shape of the VIX futures curve, they cannot perfectly replicate the VIX index. When the market is in contango over the first two expiry dates, i.e. when near term VIX futures are cheaper than longer term VIX futures, long ETNs will suffer from rolling each day a part of the positions at a higher price.

On the contrary, when the futures curve is in backwardation, i.e. near term contracts are higher than long term contracts, the ETN will benefit from rolling its positions.

The graph shows the evolution of the VIX spot vs the VXX. It shows that in the long-term the VXX has strongly suffered from the costs of roll due to the contango (80% of the time).

![Evolution of VIX spot vs VXX ETN](image)

#### Other ETPs

Further ETPs offer other strategies such as short VIX exposure or long leveraged exposure. These products have become increasingly popular.

The XIV and XXV aim to provide the inverse performance of the VIX short-term futures index. These ETNs will benefit from retracements of the VIX spot and futures, the contango in the market when applicable, but will suffer major drawdowns when volatility spikes.

The UVXY ETF aims to offer twice the performance of the VIX short-term futures Index. By doubling the exposure, investors aim to double the returns.
However, replication of short or leveraged exposure is more complex than for the straight long exposure and the associated costs cap the performance below the desired return.

ETPs overall are more suitable for **tactical use** rather than for buy and hold purposes. While the exposure to short-term maturities provide a high reactivity, the **costs** impact the ability to generate positive returns in the long run.

**Building Volatility exposure : summary PROs & CONs**

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Liquidity</th>
<th>Bid-Ask Spread</th>
<th>Counterpart</th>
<th>Implementation</th>
<th>Investment Horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Listed Vanilla Options</td>
<td>High</td>
<td>Medium</td>
<td>Market</td>
<td>Complex</td>
<td>Flexible</td>
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<tr>
<td>Variance Swaps</td>
<td>Medium</td>
<td>High</td>
<td>Bank</td>
<td>Easy</td>
<td>Flexible</td>
</tr>
<tr>
<td>VIX Futures</td>
<td>High</td>
<td>Low</td>
<td>Market</td>
<td>Intermediate</td>
<td>Tactical</td>
</tr>
<tr>
<td>VIX ETPs</td>
<td>High</td>
<td>Very Low</td>
<td>Issuer (if ETN)</td>
<td>Easy</td>
<td>Tactical</td>
</tr>
</tbody>
</table>

**Quiz**

Which instrument would you use to hedge an upcoming “known unknown” event (e.g.: rate decision, elections, referendum)?
V. Option Valuation – Understanding the Greeks

The price of an option, also called premium, can be influenced by several factors. To trade options, it is thus essential to understand which parameters affect the price of an option and what impact the change of one parameter has, in order to appreciate the risk/reward of an option strategy.

In this chapter, we will explain the “Greeks” also called “risk sensitivities”, which indicate how an option is exposed to the evolution of the different factors. Several Greeks exist but we will focus on the most important: Delta, Vega, Theta, Rho and Gamma.

As a reminder the price of an option has two components:
- The intrinsic value which depends solely on the underlying price (S) in relation to the strike price (K): \([S-K]\) for a call option and \([K-S]\) for a put option.
- The time value which is the value of the remaining time until expiry of the option.

In this chapter we use examples of options on equity indices. The option premium is expressed in index points.

### First order Greeks

First order Greeks give the sensitivity of the option price for a variation of one parameter, while all other variables remain unchanged.

#### Delta: How does the premium of an option vary when the underlying price changes?

The Delta \(\delta\) measures the average variation of the option’s price \(\Delta P\) with respect to an equivalent upward and downward change of the underlying price \(\Delta S\): \(\delta = \frac{\Delta P}{\Delta S}\). It is expressed in \%.

The graphs below illustrate how the option value changes as the Euro Stoxx 50 moves, for both 1-yr ATM call and put options with a strike price at 2800.

![Premium of a call option](image)

![Premium of a put option](image)

Source: Bloomberg as of 30th June 2016, options on the Euro Stoxx 50

Example of the call: if the Euro Stoxx 50 increases by 100 points to 2900 (\(S^+\)), the option premium will rise from 200 to 249 points (\(P^+\)) and if the index drops 100 points to 2700 (\(S^-\)), the premium will decrease to 156 points (\(P^-\)).

The Delta of the call option for a 100-point move in the Euro Stoxx 50 is: \(\delta = \frac{\Delta P}{\Delta S} = \frac{P^+ - P^-}{S^+ - S^-} = \frac{249 - 156}{200} = 46.5\%\).
The graph on the right shows the behaviour of the Delta for the call and put option subject to the value of the underlying index.

As the call option becomes increasingly in-the-money (underlying rises), the value of the option converges toward its intrinsic value. \( \Delta P \) gets increasingly similar to \( \Delta S \) and the option value moves almost like the underlying. The Delta rises towards 100%. Conversely, as the call option becomes further out-of-the-money (underlying decreases), the Delta drops to 0 and thus \( \Delta P \) as well.

Note that call and put options have opposite Delta levels. The Delta of a call option varies between 0 and 100%, whereas the Delta of a put option varies between -100% and 0.

At-the-money call and put options have respectively a Delta of approx. 50% and -50%. The Delta also gives the probability that an option will expire in-the-money. A call option with a 50% Delta means that the option has a 50% chance of expiring with the underlying above the strike price.

**Vega: Impact of the implied volatility on the option premium**

The Vega measures the option’s sensitivity (\( \Delta P \)) to an implied volatility variation (\( \Delta \sigma \)). Please note that the Vega is the same for a call and a put option with the same strike price and maturity.

When implied volatility increases, the option value goes up and conversely. Hence option holders benefit from rising implied volatility.

The left graph illustrates the relation between implied volatility and the premium of an ATM call option. For example, if the implied volatility rises by 1 point (e.g. from 21% to 22%), the option premium will increase by 11 index points.

The right graph shows the Vega for options with different maturities subject to the value of the underlying. The longer the time to expiry, the higher the Vega, as the time value is sensitive to changes in volatility. The evolution of the Vega according to the spot price is represented by a bell shape. The Vega is the highest for ATM options.
. **Theta: How much value does the option lose each day?**

The Theta measures the option's sensitivity ($\Delta P$) to the passage of time ($\Delta t$). It is the value lost the next day all things being equal. Theta will be the "enemy" of option holders, but option sellers will benefit from it.

Importantly, the Theta is not linear! As time passes, the value lost each day increases. The greatest loss to time decay is in the last weeks of the option’s life. Indeed, one day has a greater relative importance when there are only a few days left compared to longer periods.

The Theta is the same for call and put options with the same strike price and maturity.

The graph to the right illustrates the Theta of an option as a function of the remaining time to expiry.

For example, when the option expires in 6 months, the option premium declines by 0.5 points per day.

Source: Bloomberg, 30th June 2016, options on the Euro Stoxx 50

. **Rho: Impact of interest rate changes on option value**

Rho measures the option’s sensitivity ($\Delta P$) to a change in the interest rate ($\Delta r$). The closer an option is to its expiry date, the lower will be its sensitivity to an interest rate move.

The premium of call options rises as interest rates increase. Inversely the premium of put options decreases.

For example, if a 3-month call option has a Rho of 5, then for a 1% increase/drop in interest rates, the premium of the option increases/drops by 5 index points.

Source: Bloomberg, 30th June 2016, options on the Euro Stoxx 50

### Second order Greeks

Second order Greeks measure the change of first order Greeks relative to a change in another variable. Several second order Greeks exist (Volga, Vanna, …) but we will focus on the Gamma ($\gamma$).

The Gamma (in %) measures the Delta’s sensitivity ($\Delta \delta$) for a 1% underlying equity movement ($\Delta S/S$): $\gamma = \frac{\Delta \delta}{\Delta S/S}$.

Whereas the Delta can be seen as the speed of the option price movement, the Gamma is the acceleration. The Gamma is an important measure of convexity i.e. the non-linearity of the price changes.

The Gamma is the ally of the option holder, whereas it is a risky element for the seller.

Unlike the Delta, the Gamma will be the same for call and put options with the same strike price and maturity.

Let’s look at a 1-yr call option on the Euro Stoxx 50 with a strike at 2800. If the index increases by 100 points to 2900, the Delta will rise to 50.8%. If the Euro Stoxx 50 falls by 100 points to 2700, the Delta will decrease to 38.7%.

The Gamma of the call option for a 3.5% move (200/2800) in the Euro Stoxx 50 is: $\gamma = \frac{\Delta \delta}{\Delta S/S} = \frac{50.8%-38.7%}{200/2800} = 1.6%$.
As the underlying increases, the option becomes more exposed to the upside (Delta rises). Conversely, as the underlying decreases, the option becomes less exposed to the downside (Delta declines).

The convexity of the call option is reflected by the fact that the investor benefits more from the upside potential of the underlying than he is impacted by a downward move.

Source: Bloomberg as of 30th June

The Gamma is the highest when the option is at the money. It decreases as the option moves in or out-of-the-money. Options deeply in or out-of-the-money have a Gamma close to 0.

For ATM options, the Gamma increases as the time to expiry shortens.

Evolution of the Delta of a call option subject to the level of the underlying (Euro Stoxx 50)

Evolution of the Gamma for ATM options with different maturities, subject to market moves

Greeks measure the impact of one parameter, with all other things being equal. However in reality, several parameters will move together. It is thus essential to analyse the Greeks in light of multiple simultaneous changes.

Quiz

Which of the five “Greeks” discussed in this chapter is not a Greek letter?
VI. Building volatility exposure (Vega) with options

In this chapter, we will explain how to build a volatility exposure by investing in equity index options. The purpose of the investment: e.g. tactical short term hedge of an identified risk (such as the Brexit referendum), strategic portfolio diversification, or generation of absolute returns will guide investors on fundamental questions such as long versus short exposure, calibration of option strategy, level of Vega exposure, choice of underlying, etc...

As an example we will show how to build a long Vega exposure with listed vanilla options on the Euro Stoxx 50 and assume its spot level to be at 3000. As mentioned in Chapter 4 of this Volatility Academy, the equity and interest rate exposure can be hedged with futures.

In order to build a Vega exposure, an investor needs to answer the following questions:
Which maturity should be used?
How many different options/strikes are appropriate?
What should be the weight of the different options/strikes?

■ Which maturity?

The maturity of the option strategy will depend on:
- The purpose of the investment
- The liquidity of listed options: the liquidity is the highest between 0 and 24 months
- The reactivity of the implied volatility: the shorter the maturity of options, the higher the reactivity
- The level of Vega: the shorter the time to expiry, the lower the Vega of an option
- The costs of replication: the shorter the time to expiry, the higher the cost of roll and the greater the loss to time decay

Short term risks are best hedged with exposure to highly reactive short term options. For strategic positioning and mid-term absolute returns, the 1-year implied volatility offers a good balance between liquidity, reactivity and replication costs.

Not all the maturities are available on the market. For example, listed options on the Euro Stoxx 50 have the following expiry dates: 3 next months, 3 next quarters (among March, June, September and December), 3 next semesters (June and December) and 9 next years (December).

As of today, in order to obtain for example an average maturity of 1 year one would have to buy options with expiry dates in June ‘17 (10 months) and in December ‘17 (16 months). For simplicity, we will focus on the June ‘17 expiry in the following explanations.

■ How many different options/strikes?

The evolution of the Vega according to the spot price is represented by a bell shape. The Vega will be the highest when the forward level of the underlying index equals the strike (the forward level depends primarily on the spot level, the dividend yield and the risk free rate).

As the spot (forward) price moves away from the strike price, the Vega decreases. A single option is therefore not satisfactory to build a reasonably stable Vega exposure.

Source: Bloomberg calculations as of 16/08/16
The combination of several options with different strikes allows to obtain a more stable and less concentrated Vega. The graph below shows the combined Vega of a 5-option strategy consisting of long Put 80%, Put 90%, Call 100%, Call 110% and Call 120%.

As a comparison the orange curve shows the Vega of a single call option strike 3000 calibrated so that the at-the-money (“ATM”) Vega equals the one of the combination of the 5 options.

![Graph showing the combined Vega of a 5-option strategy](image)

Source: Bloomberg calculations as of 16/08/16

If the index drops 15% the Vega of the single option will be halved, whereas for the combined strategy it will reduce by only one quarter.

**Which weight should be allocated to each strike?**

Thus to maintain a more stable Vega, it is imperative to build a portfolio with several strikes (e.g.: between 80% and 120%). The volatility skew should influence the weight of each strike as the level of implied volatility varies with the strike. Indeed implied volatility of out-of-the-money (“OTM”) equity put options is normally higher than for OTM call options (see chapter 3). Therefore, the equity volatility skew is most of the time positive:

- If the skew is high in historical terms, investors can for example underweight OTM puts and overweight OTM calls, in order to avoid overpaying volatility on put strikes.
- If the skew is relatively low, investors can overweight OTM puts and underweight OTM calls.

Thus, the choice of the strikes and the weight allocated to each option are a compromise between the stability of the Vega and the level of the skew.

**The need for adjustments**

Investing in volatility via options requires active management. Indeed, the Vega moves with:

(i) **The underlying market level.** As the spot price moves, strikes need to be readjusted. If the market drops by 10%, a call option which was ATM becomes OTM and its Vega decreases.

(ii) **The passage of time.** The Vega decreases as the time to expiry shortens. If the investor wants to be exposed on average to a 1-year maturity, he will need to roll his positions regularly.

The cost of rolling the options depends on the term structure of volatility. If the term structure is steep, which is the case in normal market conditions, longer maturities are more expensive leading to costs of roll. On the contrary, when the term structure is inverted, e.g. during periods of market stress, rolling options to longer maturities will be profitable as they are cheaper in terms of volatility.
Maintaining a 1-year implied volatility exposure, can typically be achieved by investing in options with maturities between 9 to 15 months. Looking at the volatility spread between these two maturities is a good proxy to determine the cost of roll. The graphs below feature the spread between 15-month and 9-month ATM options for the S&P 500 (left graph) and the Euro Stoxx 50 (right graph).

The cost of roll differs according to the underlying. It is generally higher on the S&P 500 than on the Euro Stoxx 50. For the latter the spread used to be positive (i.e. a cost) in 2013 and 2014, whereas it became on average negative since 2015, meaning that investors can now roll their options at favourable terms.

(iii) The level of implied volatility. The Vega moves according to the level of implied volatility. In case of market stress, investors’ perception of risk increases leading to an increase of the implied volatility and of the Vega for OTM options.

In summary, maintaining the Vega of an options portfolio at a desired level requires constant monitoring and frequent readjustments as markets move and time passes by.

- How much cash is spent on option premia?

When an investor buys an option, he will pay a premium. This price depends primarily on the underlying market level, the level of implied volatility, the strike of the option and the time to maturity.

Let’s take the example of a fund of €100 million, which targets a Vega of 1 (i.e. €1 million) and a 1-year maturity:
- The strategy implemented combines 5 options expiring in June ’17 and 5 options expiring in December ’17 on the Euro Stoxx 50. At the outset the options are either OTM or ATM: Put 2400, Put 2700, Call 3000, Call 3300 and Call 3600.
- The strategy is equally-weighted, meaning that the Vega of each option is €100,000.
- The premium spent upon implementation for this trade is €11.6m, i.e. around 12% of the fund.

As the market moves, the options implemented in the fund are adjusted and the portfolio will include both in- and out-of-the-money options. Hence depending on the market environment, for a Vega of 1, investors may use up to 40% of the portfolio on option premia. On average the level is around 25%.

Holding a portfolio of options may lead to other costs, primarily the time decay. The next chapter will focus on this topic and we will discuss how to best mitigate these costs.

- Quiz

What is the average Vega outstanding on listed options on the Euro Stoxx 50 and on the S&P 500?
VII. Delta hedging, Gamma trading and Theta management

In order to build exposure to equity market volatility via options (discussed in the previous chapter) the equity and rate risk need to be hedged. In this chapter, we will discuss how to hedge the risk of underlying price movements inherent to an option (Delta hedging). Furthermore we will discuss the rationale for dynamically managing this hedge (Gamma trading). Finally we will explain the relation between Gamma trading and Theta, i.e. the time decay.

Let us remind you the following definitions, for which we assume that all other parameters are respectively unchanged:
- The Delta measures the sensitivity of the option premium to an up/downward move of the underlying’s price;
- The Gamma measures the Delta’s sensitivity for a 1% movement of the underlying;
- The Theta measures the option’s sensitivity to the passage of time. It represents the value lost the next day for an option buyer.

**Delta hedging and Gamma**

The option premium varies as the price (value) of the underlying asset changes. The impact of underlying price moves can be hedged by implementing a Delta neutral strategy, i.e. by shorting or buying (subject to the option) directly the underlying (in case of a single stock option) or futures on the underlying (in case of an index option).

A long call position has a positive Delta, therefore Delta hedging will imply **selling the underlying**. It is the opposite for a long put position. The Delta gives the **hedge ratio**, i.e. the proportion of the underlying to be sold (or bought) to achieve an overall Delta neutral position.

We will use the example of Euro Stoxx 50 at-the-money (ATM) call options (strike and spot both at 3000) expiring in Sept'17. The Delta at 46.8% implies that Delta hedging a long position of 100 options requires to sell 46.8 futures contracts (as an index point has the same value for options and futures for the Euro Stoxx 50, i.e. € 10 per point).

A portfolio, which is long 100 call options and short 46.8 futures should thus remain unaffected to moves of the Euro Stoxx 50 index. However the **equity sensitivity of options is not constant**. Indeed the Delta varies as the underlying price moves, as reflected by the Gamma.

All other parameters being equal, changes in the Delta mean that:
- If the Euro Stoxx 50 rises, the Delta of the call options rises and thus the option premium will increase more than the percentage of the initial Delta.
- If the Euro Stoxx 50 recedes, the Delta of the call options reduces and thus the option premium will decrease less than the percentage of the initial Delta.

Source: Bloomberg as of 30/09/2016

Once the underlying index has moved the strategy is no longer Delta neutral, as the Delta itself evolved. Adjusting the Delta hedge, i.e. adjusting the proportion of shorted futures, and thus engaging into Gamma trading is therefore necessary to reestablish the Delta neutral position.
### Gamma trading

Gamma trading (or dynamic hedging) involves adjusting the Delta hedge of an options portfolio to the changes of the option’s Delta caused by the changes of the underlying asset. It is called Gamma trading as the Gamma reflects the sensitivity of the Delta to changes of the underlying.

In our example, if the Euro Stoxx 50 increases by e.g. 1.0%, the Delta of the call options increases from 46.8% to 48.4%. Indeed the Gamma of these ATM options is 1.6%. In order to preserve the overall Delta neutral position at the new spot level of the index, 1.6 (= (48.4% - 46.8%) x 100) additional futures contracts need to be sold at the new spot level.

If then the index returns to its initial level, the Delta will revert to its initial (lower) level and therefore we would be short too many futures and would need to buy back 1.6 futures contracts.

The chart on the right illustrates this example.

The two adjustments of the hedge position allow to crystallise a P&L of €480 (= (3030 – 3000) x €10 tick size x 1.6 futures contracts). This does not take into account any transaction costs.

**Example of Delta hedge readjustments**

![Chart illustrating Gamma trading](chart.png)

Source: Amundi as of 30/09/2016

### Gamma trading: summary of the different steps

<table>
<thead>
<tr>
<th>Long Options</th>
<th>Short Options</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Long Call</strong></td>
<td><strong>Short Call</strong></td>
</tr>
<tr>
<td>Sell Futures on Underlying</td>
<td>Buy Futures on Underlying</td>
</tr>
<tr>
<td><strong>Long Put</strong></td>
<td><strong>Short Put</strong></td>
</tr>
<tr>
<td>Buy Futures on Underlying</td>
<td>Sell Futures on Underlying</td>
</tr>
</tbody>
</table>

**You earn money through adjustments**

**You lose money through adjustments**

Source: Amundi

For an option holder, Delta hedge readjustment (Gamma trading) allows to make a benefit whether the price of the underlying goes up or down. It is the opposite for an option seller.
Theta impact on options portfolio, correlation between Theta and Gamma

The Theta is the amount of option premium lost on the next day. Going back to the example of Euro Stoxx 50 at-the-money (ATM) call options (strike and spot both at 3000) expiring in Sept’17, when using an implied volatility of 25% the option premium (in index points) would be 232.45 and the Theta at -0.41. In other words, everything being equal, from one day to the next the value of the option would drop by 0.41 index points.

As shown in chapter 5, the negative Theta impact on a long option position increases as time to expiry shortens, i.e. for a long option position, all things being equal the loss of time value increases day after day and becomes exponential at the end.

At constant maturity, the Theta also increases with the implied volatility (IV), which is not surprising as IV affects the time value of an option, but not its intrinsic value (see chapter 2).

The Gamma is also impacted by the time to maturity and the IV. As the expiry date gets closer the Gamma rises. However, it reduces as IV goes up. Indeed the Delta will be higher for a higher IV, but its sensitivity to IV is lower.

Gamma trading potentially offsets Theta

In the following example (based on a portfolio long 100 ATM Euro Stoxx 50 call options and short 46.8 Euro Stoxx 50 futures) we show the impact of the passage of time (one day) and a daily index variation. Whereas the passage of time causes a loss of time value (due to long option position), an index variation allows to generate some P&L via the Gamma trading.

We used an implied volatility of 25% to price the option. When the spot level equals the strike (= 3000), the premium per option amounts to 232.45 index points.

Payoff of Delta neutral position (Long 100 Calls, Short 46.8 Futures) after 1 day subject to index variation

<table>
<thead>
<tr>
<th>Realised Volatility (annual.)</th>
<th>Daily Index Variation</th>
<th>P&amp;L Long Call €</th>
<th>P&amp;L Short Futures €</th>
<th>Overall Payoff €</th>
</tr>
</thead>
<tbody>
<tr>
<td>48%</td>
<td>-3%</td>
<td>-40370</td>
<td>+42120</td>
<td>+1750</td>
</tr>
<tr>
<td>32%</td>
<td>-2%</td>
<td>-27540</td>
<td>+28080</td>
<td>+540</td>
</tr>
<tr>
<td>16%</td>
<td>-1%</td>
<td>-14210</td>
<td>+14040</td>
<td>-170</td>
</tr>
<tr>
<td>0%</td>
<td>0%</td>
<td>-410</td>
<td>0</td>
<td>-410</td>
</tr>
<tr>
<td>16%</td>
<td>+1%</td>
<td>+13870</td>
<td>-14040</td>
<td>-170</td>
</tr>
<tr>
<td>32%</td>
<td>+2%</td>
<td>+28620</td>
<td>-28080</td>
<td>+540</td>
</tr>
<tr>
<td>48%</td>
<td>+3%</td>
<td>+43870</td>
<td>-42120</td>
<td>+1750</td>
</tr>
</tbody>
</table>

Source: Amundi, 30/09/2016

It appears that when the realised volatility (RV) exceeds the implied volatility of 25%, the P&L achieved through the Gamma trading offsets the loss of time value.
The graph below shows the evolution of the P&L of the Delta hedged portfolio described above. It includes the impact of the Theta and of the Gamma trading. Please note, for simplicity, the calculations assume constant implied volatility, constant Theta and constant Gamma over the 4 day period despite the index moves.

- On Day D: the Delta neutral position (long 100 calls and short 46.8 futures) is implemented.
- D+1: the Euro Stoxx 50 rises 2% to 3060; the Delta rises 3.2 points and therefore 3.2 additional futures are sold. As RV exceeds IV the P&L is positive at € 540.
- D+2: the index rises another 30 points. The short position is increased by further 1.6 futures. As RV is less than IV the P&L on the day is negative at € -170.
- D+3: the index does not move at all. The Delta hedge does not need to be readjusted. The Theta impacts the P&L with a loss of € 410.

The total P&L from implementation to D+3 is negative and stands at € -40. This is explained by the fact that the volatility realised over the 3 days (16%) is below the implied volatility paid (25%).

Evolution of a Delta hedged portfolio subject to index variations over 4 days (including Theta impact)

<table>
<thead>
<tr>
<th>Payoff €</th>
<th>D+1 Payoff</th>
<th>D+2 Payoff</th>
<th>D+3 Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>D+1</td>
<td>D+2</td>
<td>D+3</td>
</tr>
<tr>
<td>500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2940</td>
<td>D</td>
<td>D+1</td>
<td>D+2</td>
</tr>
<tr>
<td>3000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3060</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3120</td>
<td>D+3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Amundi, 30/09/2016

From theory to real life

In real life, option buyers normally pay a risk premium, i.e. a premium of implied over realised volatility, justified by the risk taken by option writers (or issuers). From the perspective of an option buyer, if the RV turns out to be lower than the IV that he paid, he will have “over-paid” the option. Therefore Theta is the enemy of option holders and the ally of option writers (pursuing for example carry strategies).

The more the price of the underlying asset/index fluctuates, i.e. the higher the realised volatility, the more the Delta will vary and the higher the returns generated with Gamma trading will be. In case of strong intraday volatility, the Delta hedge can be adjusted several times a day.

Quiz

Regarding a long position in medium term call options, what other solution can, subject to market conditions, allow to mitigate the Theta?

Selling short dated options, as they can feature a higher Theta and a lower Vega than longer dated options.
VIII. Portfolio distortion subject to implied volatility, passage of time and market moves

In the previous chapter, we have seen that when realised volatility exceeds implied volatility, Gamma trading allows to offset the loss of time value (Theta) of a long option position. In this chapter, we will consider another way to neutralise the Theta, which consists in selling short dated options. We will also show how a portfolio, including long and short option positions, and the Greeks evolve i) as implied volatility changes, ii) as time goes by and iii) as the underlying moves away from the strike price.

The examples used in this chapter are based on a portfolio, which replicates a calendar strategy, i.e. which is long 1-year call options and short 1-month call options, both on the Euro Stoxx 50 with a strike and spot level at 3050 upon implementation.

Neutralising the negative Theta of 1-year options by selling short dated options

A portfolio, which is long 1-year call options is Vega positive, Gamma positive and Theta negative. One way to neutralise the Theta is to sell short dated options. As seen in the chapters 5 and 7, short dated options feature a higher Theta and Gamma, and a lower Vega. Thus, being short 1-month options means being Gamma negative, Theta positive and slightly Vega negative.

We consider three implied volatility (IV) scenarios according to which we calculate how many 1-month options need to be sold in order to hedge the Theta: 1) steep, 2) flat and 3) inverted term structure.

The graph on the right shows the IV assumptions for each scenario and how many 1-month options need to be sold to neutralise the negative Theta of the long 100 1-year call options. As the term structure flattens or becomes inverted, fewer 1-month options need to be sold to achieve an overall Theta at (almost) zero.

In case of the steep term structure selling 37 1-month options allows to quasi neutralize the Theta, vs 28 in case of a flat term structure and 23 when it is inverted.

The table below summarises the Theta, Gamma and Vega for the 3 scenarios taking into account the different number of options.

<table>
<thead>
<tr>
<th>IV Term Structure</th>
<th>Theta</th>
<th>Gamma</th>
<th>Vega</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short 1-month Call</td>
<td>Long 1-year Call</td>
<td>Short 1-month Call</td>
<td>Total</td>
</tr>
<tr>
<td>Steep</td>
<td>-32.1</td>
<td>31.7</td>
<td>-0.4</td>
</tr>
<tr>
<td>Flat</td>
<td>-36.3</td>
<td>36.0</td>
<td>-0.2</td>
</tr>
<tr>
<td>Inverted</td>
<td>-40.4</td>
<td>39.4</td>
<td>-0.9</td>
</tr>
</tbody>
</table>

Source: Amundi, Bloomberg as of 19th October 2016

Hedging the Theta does not materially impact the Vega. However it does reduce the Gamma and pushes it potentially into negative territory, meaning that dynamic Delta hedging (i.e. Gamma trading) becomes costly (as shown in the previous chapter).
Impact of changes in implied volatility on P&L of calendar strategies

The following calculation highlights the P&L impact that a change of IV can have on the calendar strategies shown above.

In our example, a shift from a steep term structure to an inverted term structure means that:
- On the long position, the strategy benefits from the increase in the 1-year IV from 20% to 25%.
- On the short position, the strategy suffers from the increase in the 1-month IV from 15% to 30%.

The P&L in index points is calculated as follows:

\[
\text{P&L} = (\text{number of 1yr options} \times \text{Vega of 1yr options} \times \text{shift in 1yr IV}) + (\text{number of 1m options} \times \text{Vega of 1m options} \times \text{shift in 1m IV})
\]

\[
= 100 \times 11.556 \times (25 - 20) - 37 \times 3.527 \times (30 - 15)
\]

\[
= 5783.0 - 1957.5 = 3825.5
\]

The P&L is less than "the overall Vega exposure times 5 volatility points", as the negative volatility exposure on the shorted options results in a more significant loss, as on the short end volatility rises 15 points.

In this example the shorted options attenuate the P&L of the long Vega position. In case of excessive inversion of the term structure, the total P&L could be even lower.

Impact of the passage of time on Theta and Gamma

The following analysis aims to shed some light on how Theta and Gamma evolve with the passage of time, assuming all other parameters remain unchanged.

Upon implementation, all three calendar strategies have been calibrated to feature a Theta at (almost) zero.

As time passes, the Theta of each of the three scenarios becomes positive. Indeed, as time to expiry shortens, the Theta of the short dated options increases more in absolute terms than for longer dated options.

Selling short dated options means that we benefit from this exponential increase of the Theta, which more than compensates the loss of time value on the 1-year options.

Upon implementation of the three calendar strategies, the Gamma is either negative (steep term structure), at zero (flat) or positive (inverted). More importantly as represented by the graph on the right, in all scenarios the Gamma will reduce further as time passes and sooner or later reach negative territory.

The Gamma increases when the time to option expiry reduces. Thus the impact on the negative Gamma of the sold options increasingly outweighs the impact on the positive Gamma on the long option position.

Source: Amundi, Bloomberg as of 19th October 2016
As a consequence, the Theta of the calendar strategies rises, as time goes by, though this positive impact is mitigated by the increasingly negative Gamma, leading to more costly Delta hedge readjustments.

**Impact of underlying movements on Theta and Gamma**

Last but not least we will now explore how the Theta and Gamma evolve with market movements. For this we assume again that all other parameters remain unchanged.

The Theta reduces as an option is increasingly in- or out-of-the-money. This impact is stronger i) as the implied volatility is lower and ii) as the maturity is shorter.

In the calendar strategies the positive Theta of the shorted 1-month options quickly moves to zero as the spot level moves away from the strike. This impact is the strongest in the steep term structure scenario. However, as the negative Theta of the long 1-year options does not converge towards zero as quickly, it drags the overall Theta of the calendar strategy into negative territory, whichever way the underlying moves. This is shown on the graph on the right.

For very significant market movements the Theta of the longer dated options and hence of the overall portfolio will also converge to zero.

The right graph illustrates the evolution of the Gamma according to the underlying index variations. Similarly to the Theta, the Gamma is the highest when an option is at-the-money and it reduces as the option is increasingly in- or out-of-the-money. Furthermore, the shorter the maturity is, the higher the Gamma will be. However the Gamma of the shorter dated options converges more quickly towards zero (see last graph of chapter 5).

As a consequence, the graph showing the impact of underlying price variations on the Gamma almost mirrors the one for the Theta.

In the calendar strategies, the Gamma of the short dated options will quickly move to zero as the underlying index value changes. The Gamma of the longer dated options being less reactive to changes of the underlying, will not converge towards zero as quickly and thus outweigh the Gamma of the sold short dated options.

This shows that the correlation between Theta and Gamma, shown in the previous chapter for a single option, is preserved for a portfolio of options, even as the market moves.
## Conclusion

We have shown the respective consequences i) of changes in implied volatility levels, ii) of the passage of time and iii) of movements of the underlying asset on a portfolio including long and short option positions with different maturities. These consequences can differ between a single option (explained in chapter 5) and a multi-option portfolio. The individual impacts highlight that a portfolio of options requires constant readjustments in order to preserve the desired level for the various Greeks.

In real life, all these parameters move at the same time. Furthermore, the skew, which we have omitted for simplification purposes in our explanations above, impacts the level of IV as the moneyness of an option changes. These multiple simultaneous impacts render the readjustments even more complicated. Hence managing a multi-option portfolio requires deep understanding of how options work as well as appropriate and robust tools to monitor and manage it on a daily basis.

## Quiz

Can a multi-option portfolio, with an overall Vega at zero generate a positive/negative P&L, if volatility levels move?

Under a constant volatility assumption – as assumed by Black & Scholes – the answer is no. In real life, volatility levels vary according to the maturity (term structure) and the moneyness of the option (skew). Hence in a portfolio, the different options may not be impacted in the same way, thus creating a positive/negative P&L.
IX. Products impacting the volatility landscape

In the previous chapter, we saw that a portfolio of options is distorted by market movements, the passage of time and changes in implied volatility (“IV”). Implied volatility is itself impacted by:

- changes in realised volatility,
- risk aversion leading to demand for protection, or risk appetite driving demand for upside exposure, and
- flows from products directly and indirectly affecting implied volatility (IV) levels.

These factors do not necessarily impact the entire volatility surface in a homogenous way, and will therefore often distort the skew (IV according to the strike) and/or the term structure (IV by maturity).

In this chapter, we will focus on some products, which affect the volatility environment:

- Auto-calls, as an example of structured products,
- Various ETNs/ETFs (Exchange Traded Products) using VIX futures,
- Target volatility products, i.e. essentially balanced multi-asset portfolios with a volatility target.

We do not aim to describe exhaustively all potential products/impacts, but rather illustrate the variety of factors, which can affect the volatility environment.

### Auto-callable products

In the low rate environment investment banks devised structured products, mostly sold to retail investors, which allow to generate returns. The fundamental idea is to sell options on indices to collect the option premium, which allows to pay a coupon. A variety of structures have emerged over time with slightly different pay-off profiles. We have chosen a standard and rather simple version using Asian options for our illustration purposes.

The graph on the right illustrates the pay-off of a 5-year auto-callable product:

- As long as the underlying index remains within a predetermined range (60-100% in our example) the coupon cumulates. It will be paid out in case the product is called prior or at maturity.
- If the underlying equity index reaches the upper limit (100%) at pre-defined dates the structured product is automatically recalled. The investor receives the principal amount and the cumulated coupons but not the outstanding ones.
- If the product matures without being called, i.e. the index does not exceed the range, the investor gets the principal back but the coupons are forfeit.
- If however the index drops below a predetermined knock-in level (e.g. 60%) the investor becomes long the underlying equity index and is exposed to equity losses since issuance.

These products have an impact on long-term implied volatility as investors buying auto-calls are short volatility (selling options) and the issuing banks long volatility. The latter need to hedge their volatility exposure by trading mostly vanilla options in the market.

- At inception of the product, the bank will sell options, to hedge its position and thus put downward pressure on IV.
- As the underlying index increases, the automatic recall becomes more likely (i.e. probability of the knock-out increasing) and the bank needs to buy back options to reduce its short position. This will put upward pressure on implied volatility.
- If the underlying slowly decreases, the probability to reach the knock-out level decreases and the bank sells more options, leading to downward pressure on IV.
- If the underlying index hits the knock-in level, the optionality of the product disappears and the bank needs to buy back all options, putting a lot of upward pressure on IV as it will buy volatility in a falling market.

As a result of the hedging by the issuing bank, auto-calls reduce the negative correlation between volatility and the underlying equity index, for small market moves within the pre-defined knock-in and knock-out levels.
Auto-calls are very successful in Japan, which has known low rates for two decades. They are now fairly common throughout Asia and in Europe. They are mostly implemented on Asian and European indices. The increasing issue amounts of these products put pressure on long term IV and on long term skew, mostly via the sale of put options.

The graph on the right shows the comparably low skew observable on the Euro Stoxx 50 and Asian indices. The skew on the S&P 500 is significantly less/not affected by such products. In turn the wider skew is due to strong demand for protection, S&P 500 put options being to some extent used as a hedge for global equity markets.

### VIX ETPs

Investors wishing to get VIX exposure, e.g. as a tactical protection strategy, can buy ETPs using VIX futures as a proxy to the spot VIX. In order to maintain a constant 1-month exposure, the ETPs will need to roll 5% of their exposure on a daily basis (assuming there are 20 trading days per month). This roll puts pressure on the first futures contract and pushes up the price of the second, thus increasing the contango of the futures curve.

The contango will increase as demand for long VIX exposure through ETPs increases, except when there is a sudden event triggering a volatility spike.

The graph to the right shows the evolution of the spread of the 2nd futures contract versus the first. It appears that this spread – reflecting the contango – is highly correlated to the amount of assets invested on long VIX strategies.

Thus in-/outflows into long VIX ETPs impact the price of VIX futures and the shape of the futures curve. Besides these ETPs, which aim to replicate the VIX, others offer a leveraged long exposure or a short exposure to the VIX.

In order to maintain a constant exposure level, the latter will need to adjust their positions as the VIX level changes. Both leverage (e.g. x2) and short ETPs have to buy additional futures as the VIX rises to maintain the leverage/exposure and sell futures as the VIX index decreases.

Hence the movements of the VIX level cause additional readjustment flows on the futures, even without any in-/outflows in the ETPs. This increases the reactivity of the futures’ prices and the realised volatility of the VIX index, even when the VIX is not particularly high.
Target Volatility Products

Target volatility products are multi-asset portfolios dynamically rebalancing their assets between risky investments (e.g. equity) and safer assets (e.g. bonds), to achieve the target volatility desired for the investment. These products sell equities when equity volatility is rising and buy equities when volatility is falling, thus exacerbating the market moves and increasing realised volatility.

Volatility control products are a driver of the large up and down swings seen in the last 2 years in the different markets and contribute to higher volatility of volatility.

The graph on the right shows the fast transitions from very low realised volatility (1-month RV below 10%) to high realised volatility (2-week RV above 20%) on the S&P 500 observed in the past 25 years. One can see a clear acceleration of such transitions as one third of them occurred in the last 2 years.

Conclusion

All the products mentioned above have a direct or indirect impact on the volatility environment: skew, term structure, realised volatility, vol of vol. The flows in and out of these products and readjustments triggered by market movements need to be taken into consideration when monitoring volatilities. While this can blur the outlook on volatilities, it can also create attractive investment opportunities as we will show in the next chapter.

Quiz

Are auto-call products good or bad for buyers of long-term volatility?
X. Volatility Investment Strategies

This final chapter of the Volatility Academy is dedicated to the major volatility investment strategies and their purposes. Volatility strategies can be divided into those offering diversification and possibly even tail hedge protection and those allowing to enhance performance.

Strategies used as **diversification or for protection**:  
- Directional long only strategies,  
- Flexible Directional volatility strategies.

Strategies used as **performance enhancement**:  
- Short volatility strategies:  
  - Carry strategies,  
  - Short only volatility strategies,  
  - Tactical short volatility strategies;  
- Long / short volatility strategies, in particular relative value strategies.

### Diversification/Protection Strategies

The purpose of the following strategies is primarily to enhance the overall risk return profile of a portfolio. As equity volatility is negatively correlated to the underlying equity market, a long volatility exposure is very diversifying. Indeed when equity markets suffer significant drawdowns, volatility spikes and a long volatility exposure will capture this move.

#### Long Only Volatility Strategies

As indicated in Chapter 4 some ETPs offer a **systematic long only volatility exposure**. The most common strategies are implemented on the VIX (e.g. VXX) and aim to replicate the performance of the VIX Index.

ETNs such as the VXX build the long volatility exposure by buying VIX Futures. In order to maintain a constant exposure to one month implied volatility (definition of the VIX) they have to roll a part of the portfolio on a daily basis, i.e. 5% per day assuming there are 20 trading days in one month. Due to this roll requirement they will suffer from the fact that the futures curve is approx. 80% of the time in contango. In the long run this significantly detracts performance.

In order to avoid the long term negative impact of being systematically long volatility, as is the case for the above mentioned long volatility ETNs, some investors will take a more tactical approach. Implementing a **tactical long volatility exposure** can be achieved by:  
- Using the long volatility ETPs tactically. Many investors do this, as shown by the in-/outflows reflected in the number of outstanding shares of these ETNs.  
- Actively buying volatility futures, and possibly rolling them, as is done by the ETNs.  
- Buying call options on a volatility index (e.g. on the VIX).
Using futures directly or via the ETPs bears the risk of an adverse volatility movement, with potentially high losses. When the volatility exposure is sought via call options, the risk is limited to the amount invested in option premium.

Tactical long volatility positioning requires good market timing in order to provide efficient protection. In order to tackle this concern, **signal based long volatility strategies** aim to provide the protection of a long volatility exposure only when a market disruption is anticipated, so as to avoid the negative impact of costs of roll and/or time decay when markets are in risk-on mode and the need for diversification appears lower. The signals or indicators used can be more or less sophisticated. Among the simple signals features a flattening/inversion of the volatility term structure or of the volatility futures curve. Indeed market turmoil normally comes along a rise of volatility and an inversion of the term structure. Obviously the quality of the signal will be key to the effectiveness of the strategy. The major risk is that of a false start, i.e. the signal suggests market turmoil, the long volatility exposure is implemented on somewhat higher levels but ultimately volatility recedes, as markets stabilise. In turn, if the signal does not anticipate a crisis then the strategy is simply not implemented, it does not offer protection, but it does not lose any money either.

**. Flexible Directional Volatility Strategies**

None of the pure long only strategies mentioned above is perfect. This is due to the fact that (implied) volatility is mean reverting and potentially costly to hold. **Flexible directional volatility strategies**, i.e. strategies that can be either long or short (not be mistaken with long/short arbitrage), take advantage of this mean reversion.

Whereas traditional asset classes do show a long term trend (normally upwards, e.g. S&P 500 performance) volatility is trendless and increasingly so, the longer the time horizon is (e.g. VIX). Similarly to a spring, which reverts to its natural position after having been compressed or elongated, volatility will ultimately return to its average level or regime.

On one hand this “mean reversion” implies that a constant exposure does not allow to generate performance in the long term, on the other hand it provides the certainty that low volatility will eventually rise and that high volatility will eventually recede.

With this mean reversion in mind and in line with any basic trading strategy, the idea of flexible directional volatility is:
- to buy volatility when it is low, and
- to sell or short it when it is high.

Thus, the strategy performs well:
- when implied volatility rises from low levels, as the portfolio will be positioned long volatility, or
- in periods of major market disruption, which cause volatility spikes to excessive levels, by turning short volatility to capture a volatility normalisation.

Active management of the volatility exposure can allow the generation of absolute returns and should improve performance over the long term.
Performance Enhancement Strategies

Usually volatility strategies are perceived as diversification tools. Nevertheless, they can also be used to generate absolute returns and thus enhance performance. These strategies normally feature a positive correlation with the underlying markets.

. Carry Strategies

In chapter 2 we explained that on average/structurally implied volatility is above realised volatility, reflecting the risk premium of the asset class, i.e. the fact that option sellers take significantly greater risk (i.e. that volatility surges) than option buyers, whose losses are limited to the option premium paid.

Carry strategies aim to capture this premium of implied over realised volatility by selling options or variance swaps. Regarding options, in order to avoid exposure to the underlying, a delta hedge will be implemented and managed dynamically. As a result the carry strategy captures the volatility premium reflected by the time value – normally beneficial to option sellers –, as long as realised volatility (RV) does not exceed implied volatility (IV), in which case the Gamma trading losses would outweigh the Theta gains (see chapter 7 for the relationship between Gamma and Theta).

The profitability of carry strategies is highly dependent on the evolution of IV vs RV post implementation and thus of market sentiment and risk aversion. These strategies are more risky when IV is low as the risk of a (even small) volatility increase is higher. In order to diversify risks, several carry strategies on different underlyings are often implemented.

Carry strategies can be implemented on a pure systematic basis or take into account some quantitative and/or fundamental analysis.

- Quantitative analysis will in particular focus on current/historic IV versus RV levels.
- Fundamental analysis will aim to ascertain whether external factors (macro factors, central bank action, currency impact, specific situation) could put at risk the strategy.

As an example we compare 1-month implied volatility with the actual realised volatility 1 month later for the S&P 500. As expected the spread between both volatilities reflects an average premium of 1 point of IV over RV since 2012. The average is fairly low. Though negative spreads are less frequent, they tend to be more extreme than when it is positive. In 2016 the spread was negative on 3 occasions: in January due to concerns over Chinese and global growth, in June around the Brexit Referendum and end of the Summer as markets feared a rate hike by the FED. The premium can be captured by shorting 1-month options on the S&P 500 and hedging the underlying or by selling 1-month variance swaps.

. Short Only Volatility Strategies

Some ETPs offer systematic short volatility exposure. For example the XIV or XXV ETN aim to provide the inverse performance of the VIX Index. These ETNs build the short exposure by selling VIX Futures, which they have to roll on a regular basis. While long VIX ETNs suffer from the futures curve being in contango, short ETN benefit from it.

Similarly to the carry strategy explained above, which aims to capture the risk premium, this strategy aims to capture the fact that the futures contracts with longer maturity bear more uncertainty than those with shorter maturity. The strategy will lose money when volatility spikes, which normally also causes the curve to flatten or to be in backwardation.
Tactical Short Volatility Strategies

Whereas the ETPs mentioned above adopt a systematic short only strategy, some investors will prefer a tactical short volatility exposure to simply take advantage of an expected reduction of volatility, usually after it spiked.

A tactical short volatility exposure can be implemented by:
- selling volatility futures (e.g. on the VIX) or equity options (e.g. on the S&P 500) and hedging the underlying, though in this case the risk of further rising volatility can be substantial;
- using a short volatility ETP tactically;
- buying a put option on a volatility index (e.g. on the VIX). The value of the put option will benefit from a volatility reduction. However the potential loss in case volatility rises further is limited to the amount spent on the option premium. Hence the strategy using put options benefits from the natural convexity of options.

Relative Value Strategies

Relative value strategies aim to exploit the divergences of implied volatilities between different underlyings, maturities or strikes, which can arise as a consequence of supply & demand, news flow or structured products. In the equity universe, the underlyings include primarily equity indices, sector indices or single stocks. Relative value strategies imply being long one IV and short another. Which positions are implemented depends whether one expects a widening or narrowing of the spread between both volatilities, as the underlying idea is to benefit from the mean reversion of this spread. If implemented via options, the sensitivity to price moves of the underlying is hedged. This is not required in case of variance swaps.

A variety of implied volatility divergences can be exploited to generate performance, including between:
- two assets like two equity indices: e.g. short volatility on the S&P 500 and long volatility on the Nikkei 225
- two maturities, i.e. playing the term structure by arbitraging say 3-month against 12-month IV.
- two strikes, to take advantage of the skew

Alike carry strategies, relative value trades can be purely systematic or rely on analysis:
- Quantitative analysis: appreciation whether volatility is “normal” or unusual compared to the volatility landscape, historic averages, realised volatility;
- Fundamental analysis : assuming an abnormality has been identified, what is the rationale for it to disappear (or accentuate), e.g. macro factors, central bank action, currency impact, specific situation
The chart to the right shows the 3-month IV spread between the Euro Stoxx 50 and the S&P 500.

A relative value trade will aim to benefit from a widening or narrowing of this spread. Usually one would “play” the mean reversion, unless fundamental analysis suggests that the spread will evolve differently.

For example at the time of the Brexit referendum, though the spread was already high, it rose even further to a level not seen since the financial crisis in October 2008.

In order to benefit from a reduction of the spread one will short the higher Euro Stoxx 50 volatility and buy the lower S&P 500 IV. The opposite positioning will allow to benefit from a widening of the spread.

The spread is not only impacted by the directional move of both volatilities but also by the relative amplitude, i.e. if one rises or drops more than the other.

### Conclusion

Investors seeking an equity and/or fixed income exposure must choose which strategy and market segment best matches their desired investment profile, e.g. large cap versus small caps or short dated high yield versus perpetual investment grade bonds.

Similarly volatility offers a variety of strategies, which aim to achieve different purposes. For each strategy it is important to use the adequate implied volatility or volatility mix (underlying asset, cost in absolute/relative terms, maturity, strike…) as the various strategies are not necessarily compatible with the entire range of implied volatilities.

The technicalities of investing in volatility are slightly more demanding than for traditional asset classes, as volatility is not directly investable. Therefore volatility exposure has to be implemented through dated derivative contracts. This adds some complexity (management of time decay and/or timing/frequency of rolling positions). However, when the volatility strategies are adequately used and managed, they can be extremely attractive either on a stand-alone basis or from a portfolio construction perspective.

We hope that you enjoyed reading this Volatility Academy and that it has contributed to improving your understanding of the various topics that we explored. The Volatility team at Amundi remains at your disposal and would be pleased to discuss any unanswered questions and potentially to structure tailor made solutions according to your needs.

### Quiz

Which volatility strategy have we missed in our catalogue?

© Amundi Management Team. If you have an innovative idea, we will be delighted to review your application for a job in our Volatility team.
Notes:
Past performance is not a reliable indicator of future results or a guarantee of future returns

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